# Object Oriented Backward/Forward Algorithm for Unbalanced and Harmonic Polluted Distribution Systems

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***Abstract*—The optimization in the operation of distribution electric systems has become an acute problem. In order to evaluate some essential criteria (e.g., active power losses) the computation of power flow is absolutely necessary. Taking into account the real operating conditions (unbalance, harmonics) it is of great interest for accurate steady state estimation. The existent solutions proposed in literature are complex and it is difficult to implement them in unbalanced and harmonic polluted systems. By the original adaptation of the backward/forward sweep algorithm, the authors propose a novel paradigm to analyze the propagation of asymmetries and harmonic distortions through radial electric networks.**

***Keywords- unbalanced electric networks; harmonic distortion; load flow; backward/forward; objects oriented programming***

## I. INTRODUCTION

The optimization in the operation of distribution electric systems has become an acute problem. In order to evaluate some essential criteria (e.g., active power losses) the computation of power flow is absolutely necessary. Currently, this computation considers that the system is balanced and the waveforms are sinusoidal. But it is well known that the modern distribution systems are unbalanced and, often, harmonic polluted. Thus, taking into account the real operating conditions (unbalance, harmonics) it is of great interest for accurate steady state estimation.

Basically, power flow algorithms are iterative and are based on different procedures: Gauss-Seidel, Newton-Raphson, backward/forward sweep. For distribution systems which are operated in radial configurations, the most recommended approach are backward/forward sweep based algorithms because of the small iterations number required and robust convergence [1].

Generally, the load flow calculation for unbalanced/asymmetrical electrical systems consists of the determination of steady state quantities for each phase at a time. In the case of unbalanced distribution systems operated in radial configurations have been developed more backward/forward sweep based algorithms with significant results as well as [2-5]. On the other hand, for harmonic polluted electric networks, the load flow calculation consists of the determination of steady state quantities for each harmonic component at a time. The existent solutions proposed in literature are complex and it is difficult to implement them in unbalanced and harmonic polluted systems.

Besides the data types existing in the numbers theory, the high level programming languages allow the definition of new artificial data types, for instance *abstract data types*. The term type of data designates a *set of values* (the domain of type) and a *set of operations* that can be performed with these values [6, 7]. The term *type of data* designates a *set of values* (the domain of type) and *a set of operations* that can be performed with these values. The set of operations can be divided in three subsets: (a) operations among the same type of data; (b) operations among the specified type of data and another type of data; (c) operations performed on the data itself. As an example, we can consider data belonging to the real numbers set (the domain of type). In this case, they can perform arithmetical operations with another real number (operations among the same data types), arithmetical operations with an integer number (operations among the specified type of data and another type), and the extraction of integer part (operation applied to the data type itself).

Based on these abstract data types, we propose to model the harmonic complex quantities (a set of complex quantities corresponding to each harmonic component) through abstract data types with complex parameters ($x_1$, $x_2$, …, $x_n$). In a same manner, we propose to model the three-phase harmonic complex quantities (symmetrical, asymmetrical, balanced or unbalanced) through abstract data types with three harmonic complex parameters (r, s, t); among these parameters, some operations are defined. By implementing this model, all three-phase harmonic quantities will be considered as "harmonic complex three-phase" objects. As a result, electrical engineering laws, as Ohm or Kirchhoff, are reduced to simplified expressions corresponding to fundamental harmonic component single-phase case. Consequently, we have introduced these objects on the backward/forward sweep algorithm with some particular adaptations.

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Based on the above considerations, the authors propose a novel paradigm (by the original adaptation of the backward/forward sweep algorithm) to analyze the propagation of asymmetry and harmonic distortion through radial electric systems. The proposed model has been implemented in the C++ programming language. In order to test the correctness of the proposed method, the authors have studied an IEEE system.

## II. BACKWARD/FORWARD SWEEP

To specify an abstract data type, it is necessary to indicate the two elements of the type, i.e. the domain and the operations set:

- *the domain:* is specified as a mathematical set;
- *the operations set:* any operation is described by its mathematical definition.

### A. Complex Quantities as Abstract Data Types

It is possible to represent a sinusoidal quantity through two real numbers in two manners (Figure 1):

- a *rotating vector*: with module (V) and angle/phase (θ);
- a *complex number*: with real part (a) and imaginary part (b);

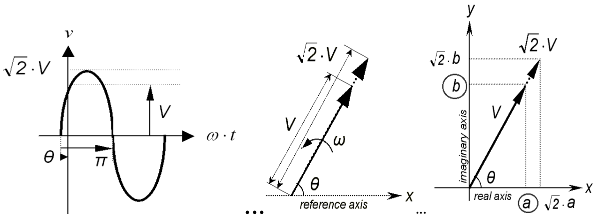

Figure 1. Electric sinusoidal quantities

Complex numbers are very frequently used in electrical engineering. They have the following domain (indicated by *C*):

$$C = \{(a,b) \mid a,b \in R\} \qquad (1)$$

### B. Harmonic Complex Quantities as Abstract Data Types

A "harmonic complex" number may represent many non-sinusoidal physical quantities (voltages, currents, impedances etc.). It is possible to represent a non-sinusoidal quantity through a set of rotating vectors (complex numbers), corresponding to Fourier series components (Figure 2).

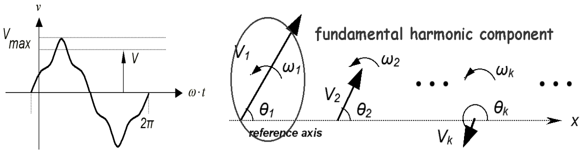

Figure 2. Electric non-sinusoidal quantities

The domain of these quantities (the harmonic complex numbers) is denoted by *HC* and specified as:

$$HC = \{(v_1, v_2, \ldots, v_i) \mid v_i \in C;\quad i = \overline{1,n};\quad n \in N^*\} \qquad (2)$$

### C. Harmonic Complex Three Phase Quantities as Abstract Data Types

Three non-sinusoidal functions can be representing as a set with systems of rotating vectors (Figure 3).

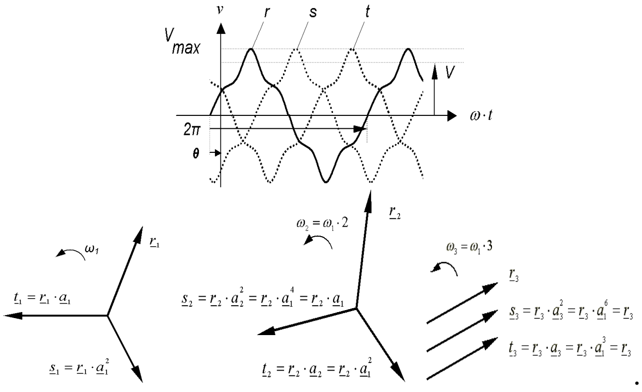

Figure 3. Electric three-phase non-sinusoidal quantities

The domain of these quantities (the complex three-phase numbers) is denoted with *HCT* and is specified as:

$$HCT = \{(r,s,t) \mid r,s,t \in HC\} \qquad (3)$$

The complete model (containing also operations sets) and further information are given in [8, 9]. With these models, the most relationships containing harmonic complex three-phase quantities can be described by simplified expressions.

In order to analyze the propagation of asymmetry and harmonic distortion through electric systems, the performing of load flow (power flow) calculation is absolutely necessary. In radial networks, the load flow calculation can be performed using a specific method, known as the backward/forward sweep. Basically, this method consists of two steps [1]:

- *backward sweep* (branch current update), where, starting from the end nodes and going towards the source node, and using the Kirchhoff's current law, the current at each load node, as well as the current flowing through its ingoing branch, are calculated;
- *forward sweep* (node voltage update), where, starting in the opposite direction, from the source node (whose constant voltage is taken as reference) and going towards the end nodes, using the Ohm's law, the voltage drop on each branch, as well as the voltage at each load node, are calculated.

In unbalanced and/or harmonic polluted power distribution systems, the above-proposed models can be introduced. Accordingly, the load flow calculation algorithm using the backward/forward sweep consists in the same steps. In the following, we will present the particular aspects of backward/forward sweep adaptation in order to incorporate power transformers and power lines.

### *D. Harmonic Three-Phase Model for Power Transformers*

It is assuming that by three single-phase interconnected transformers a three-phase transformer is constructed [2]. We will present the model for two of the most commonly used transformer connexions: Wye-Wye and Delta-Wye.

#### *1) Harmonic Three-Phase Model for Wye-Wye Connected Power Transformers*

- ***Backward sweep***

Given the currents at the secondary terminals ($I_j$) of the power transformer,

$$I_j = \{(I_R, I_S, I_T) \mid I_R, I_S, I_T \in HC\} \tag{4}$$

the aim is to calculate the currents at the primary side ($I_i$)

$$I_i = \frac{I_j}{N_{ij}} \tag{5}$$

where $N_{ij} \in C$ represents the turn ratio.

- ***Forward sweep***

Given the voltages ($V_i$) and the currents ($I_i$) at the primary terminals of the power transformer,

$$V_i = \{(V_R, V_S, V_T) \mid V_R, V_S, V_T \in HC\} \tag{6}$$

the aim is to calculate the voltages at the secondary side ($V_j$)

$$V_j = \frac{V_i - Z_{ij} \cdot I_i}{N_{ij}} \tag{7}$$

where $Z_{ij} \in HCT$ represents the harmonic three-phase impedance of power transformer.

#### *2) Harmonic Three-Phase Model for Delta-Wye Connected Power Transformers*

- ***Backward sweep***

Given the currents at the secondary terminals ($I_j$) of the power transformer,

$$I_j = \{(I_R, I_S, I_T) \mid I_R, I_S, I_T \in HC\} \tag{8}$$

the aim is to calculate the currents at the primary side ($I_i$)

$$\begin{aligned} I_a &= (I_R - I_S)/\sqrt{3} \quad I_a \in HC \\ I_b &= (I_S - I_T)/\sqrt{3} \quad I_b \in HC \\ I_c &= (I_T - I_R)/\sqrt{3} \quad I_c \in HC \end{aligned} \tag{9}$$

$$I_{abc}(I_a, I_b, I_c) \quad I_{abc} \in HCT \tag{10}$$

$$I_i = \frac{I_{abc}}{N_{ij}}, \tag{11}$$

- ***Forward sweep***

Given the voltages ($V_i$) and the currents ($I_i$) at the primary terminals of the power transformer,

$$V_i = \{(V_R, V_S, V_T) \mid V_R, V_S, V_T \in HC\} \tag{12}$$

the aim is to calculate the voltages at the secondary side ($V_j$)

$$\begin{aligned} V_a &= (V_R - V_T)/\sqrt{3} \quad V_a \in HC \\ V_b &= (V_S - V_R)/\sqrt{3} \quad V_b \in HC \\ V_c &= (V_T - V_S)/\sqrt{3} \quad V_c \in HC \end{aligned} \tag{13}$$

$$V_{abc}(V_a, V_b, V_c) \qquad V_{abc} \in HCT \tag{14}$$

$$V_j = \frac{V_{abc} - Z_{ij} \cdot I_i}{N_{ij}} \tag{15}$$

### *E. Harmonic Three-Phase Model for Real Electric Power Lines*

We will present the general model for four wires line because it has neutral conductor (Figure 4):

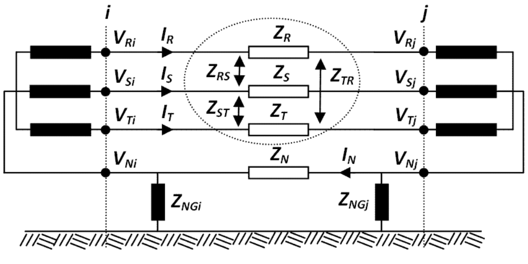

Figure 4. Model of electric power line

- ***Backward sweep***

Given the currents at the end side ($I_j$) of the power line,

$$I_j = \{(I_R, I_S, I_T) \mid I_R, I_S, I_T \in HC\} \tag{16}$$

the aim is to calculate the currents at the beginning side ($I_i$)

$$I_i = I_j, \quad I_i, I_j \in HCT \tag{17}$$

$$\begin{aligned} I_{Nj} &= \bar{I}_j = I_R + I_S + I_T, & I_{Nj} &\in HC \\ I_N &= I_{Nj} \cdot \frac{Z_{NGj} + Z_{NGi}}{Z_N + Z_{NGj} + Z_{NGi}}, & I_N &\in HC \end{aligned} \tag{18}$$

where: $I_N$ represents the flowing current through neutral conductor; $Z_N$ represents the impedance of neutral conductor and $Z_{NG}$ represents the grounded impedance.

- ***Forward sweep***

Given the voltages ($V_i$) and the currents ($I_i$) at the beginning side of the power line,

$$V_i = \{(V_R, V_S, V_T) \mid V_R, V_S, V_T \in HC\} \tag{19}$$

the aim is to calculate the voltages at the secondary side ($V_j$)

$$\begin{aligned} \Delta V_R &= Z_R \cdot I_R + Z_{RS} \cdot I_S + Z_{TR} \cdot I_T & \Delta V_R &\in HC \\ \Delta V_S &= Z_{RS} \cdot I_R + Z_S \cdot I_S + Z_{ST} \cdot I_T & \Delta V_S &\in HC \\ \Delta V_T &= Z_{TR} \cdot I_R + Z_{ST} \cdot I_S + Z_T \cdot I_T & \Delta V_T &\in HC \end{aligned} \tag{20}$$

$$\Delta V(\Delta V_R, \Delta V_S, \Delta V_T) \qquad \Delta V \in HCT \tag{21}$$

$$\Delta V = \Delta V + Z_N \cdot I_N \tag{22}$$

$$V_j = V_i - \Delta V \tag{23}$$

$$V_{Nj} = \bar{V}_j \quad V_{Nj} \in HC \tag{24}$$

## III. NUMERICAL RESULTS

The proposed algorithm has been implemented in the C++ programming language [10]. In order to test the correctness of the proposed method, the authors have studied an IEEE system for testing unbalanced radial electric networks (Figure 5) [11]:

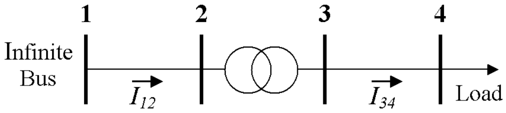

Figure 5. IEEE 4 Node Test Feeder

TABLE I. SYSTEM IMPEDANCES

| Branch | Phase | Harmonic order | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| | | 1 | | 3 | | 5 | | 7 | |
| | | ***r [Ω]*** | ***x [Ω]*** | ***r [Ω]*** | ***x [Ω]*** | ***r [Ω]*** | ***x [Ω]*** | ***r [Ω]*** | ***x [Ω]*** |
| 1-2 | R | 0.152 | 0.535 | 0.263 | 1.605 | 0.340 | 2.675 | 0.402 | 3.745 |
| | S | 0.152 | 0.535 | 0.263 | 1.605 | 0.340 | 2.675 | 0.402 | 3.745 |
| | T | 0.152 | 0.535 | 0.263 | 1.605 | 0.340 | 2.675 | 0.402 | 3.745 |
| | RS | 0.036 | 0.323 | 0.062 | 0.969 | 0.080 | 1.615 | 0.095 | 2.261 |
| | ST | 0.036 | 0.296 | 0.062 | 0.888 | 0.080 | 1.480 | 0.095 | 2.072 |
| | TR | 0.036 | 0.276 | 0.062 | 0.828 | 0.080 | 1.380 | 0.095 | 1.932 |
| 2-3 | R | 0.259 | 1.555 | 0.449 | 4.665 | 0.579 | 7.775 | 0.685 | 10.885 |
| | S | 0.259 | 1.555 | 0.449 | 4.665 | 0.579 | 7.775 | 0.685 | 10.885 |
| | T | 0.259 | 1.555 | 0.449 | 4.665 | 0.579 | 7.775 | 0.685 | 10.885 |
| 3-4 | R | 0.217 | 0.510 | 0.375 | 1.531 | 0.485 | 2.552 | 0.573 | 3.573 |
| | S | 0.221 | 0.496 | 0.383 | 1.489 | 0.494 | 2.482 | 0.584 | 3.474 |
| | T | 0.219 | 0.504 | 0.378 | 1.513 | 0.489 | 2.522 | 0.578 | 3.530 |
| | RS | 0.074 | 0.238 | 0.128 | 0.713 | 0.165 | 1.188 | 0.195 | 1.663 |
| | ST | 0.075 | 0.201 | 0.130 | 0.602 | 0.167 | 1.003 | 0.198 | 1.404 |
| | TR | 0.073 | 0.182 | 0.126 | 0.547 | 0.163 | 0.911 | 0.192 | 1.275 |

TABLE II. UNBALANCED AND NON-LINEAR LOAD (CONSTANT CURRENTS)

| Node | Phase | Harmonic order | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| | | 1 | | 3 | | 5 | | 7 | |
| | | ***Module [A]*** | ***Angle [°]*** | ***Module [A]*** | ***Angle [°]*** | ***Module [A]*** | ***Angle [°]*** | ***Module [A]*** | ***Angle [°]*** |
| 4 | R | 695.51 | -66.00 | 146.06 | -66.00 | 132.15 | -66.00 | 76.51 | -66.00 |
| | S | 1033 | 177.10 | 216.93 | -62.90 | 196.27 | 57.10 | 113.63 | 177.10 |
| | T | 1352 | 55.20 | 283.92 | -64.80 | 256.88 | 175.20 | 148.72 | 55.20 |

TABLE III. LINES CURRENTS (UNBALANCED AND NON-LINEAR LOADING)

| Branch | Phase | Harmonic order | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| | | 1 | | 3 | | 5 | | 7 | |
| | | ***Module [A]*** | ***Angle [°]*** | ***Module [A]*** | ***Angle [°]*** | ***Module [A]*** | ***Angle [°]*** | ***Module [A]*** | ***Angle [°]*** |
| 1-2 | R | 285.74 | -27.61 | 13.78 | 123.44 | 55.93 | -100.49 | 31.43 | -27.61 |
| | S | 402.68 | -149.601 | 13.00 | 109.08 | 75.09 | 21.56 | 44.29 | -149.60 |
| | T | 349.13 | 74.36 | 26.57 | -63.53 | 65.64 | 155.34 | 38.40 | 74.36 |
| 3-4 | R | 695.51 | -66.00 | 146.06 | -66.00 | 132.15 | -66.00 | 76.51 | -66.00 |
| | S | 1033 | 177.10 | 216.93 | -62.90 | 196.27 | 57.10 | 113.63 | 177.10 |
| | T | 1352 | 55.20 | 283.92 | -64.80 | 256.88 | 175.20 | 148.72 | 55.20 |

TABLE IV. NODES VOLTAGES (UNBALANCED AND NON-LINEAR LOADING)

| Node | Phase | Harmonic order | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| | | 1 | | 3 | | 5 | | 7 | |
| | | ***Module [V]*** | ***Angle [°]*** | ***Module [V]*** | ***Angle [°]*** | ***Module [V]*** | ***Angle [°]*** | ***Module [V]*** | ***Angle [°]*** |
| 2 | R | 12350.39 | 29.60 | 3.39 | 61.94 | 122.02 | 124.74 | 99.94 | -97.94 |
| | S | 12314.04 | -90.39 | 31.18 | 9.90 | 159.27 | -106.89 | 126.97 | 134.67 |
| | T | 12332.98 | 149.75 | 33.37 | -165.50 | 126.99 | 24.24 | 103.43 | 4.81 |
| 3 | R | 2290.40 | -32.40 | 69.77 | -161.12 | 139.06 | -159.47 | 112.18 | -162.34 |
| | S | 2261.58 | -153.81 | 113.64 | -158.18 | 192.78 | -39.33 | 157.17 | 83.32 |
| | T | 2213.99 | 85.18 | 154.00 | -160.67 | 253.31 | 80.00 | 204.81 | -39.23 |
| 4 | R | 2157.09 | -34.25 | 615.31 | -166.57 | 230.88 | -167.10 | 209.20 | -166.36 |
| | S | 1936.04 | -157.02 | 726.48 | -165.64 | 491.30 | -31.80 | 390.36 | 69.46 |
| | T | 1849.34 | 73.39 | 810.97 | -166.69 | 763.35 | 70.72 | 601.61 | -37.68 |

This is a radial system which contains an infinite bus, two lines, a transformer and a load. It is proposed to test the propagation of asymmetries and harmonic distortions taking into consideration the scenario: unbalanced and non-linear load, four wire and three wire lines and a Delta-Wye connected power transformer (D-GrY). On the presented case study, system impedances are given in Table I [11], source node is a *12.47 kV* line-to-line infinite bus [11], power transformer turn ratio is *12.47/4.16* [11] and the unbalanced and non-linear load data are given in Table II as constant currents.

As results, lines currents are given in Table III and nodes voltages are given in Table IV, where:

- R represents $V_{RGr}$ for wye connections and $V_{RS}$ for delta connections; (Gr represents "ground")
- S represents $V_{SGr}$ for wye connections and $V_{ST}$ for delta connections;
- T represents $V_{TGr}$ for wye connections and $V_{TR}$ for delta connections.

The aim of the initial test cases is "to make available a common set of data that could be used by program developers and users to verify the correctness of their solutions" [11] for unbalanced radial distribution systems. Thus, the systems data and the solutions do not refer to real power systems. These test cases have the unique purpose to verify the correctness of different models and implementations for unbalanced systems.

Consequently, on this paper, the harmonic three-phase line currents (given in Table III) and the harmonic three-phase voltages (given in Table IV) for "IEEE 4 Node Test Feeder" [11] have the role to demonstrate the correctness of the proposed algorithm. Also, these data can be used as a benchmark for studies with unbalanced and harmonic polluted radial systems.

## IV. CONCLUSIONS

The paper presents an original solution to perform calculus with harmonic complex three-phase quantities by using abstract data types. Related mathematical models for complex abstract data type, harmonic complex abstract data type and harmonic complex three-phase abstract data type are developed. These models can be used as objects in any program written in C++

programming language (for any calculus with complex numbers, harmonic complex numbers or harmonic complex three-phase numbers).

By implementing these objects in a backward/forward sweep based algorithm, a novel algorithm for unbalanced and harmonic polluted case (steady state) was proposed. A numerical example is given to confirm the computer implementation of proposed objects

By the original adaptation of the backward/forward sweep algorithm, the authors propose a novel paradigm to analyze the propagation of asymmetry and harmonic distortion through radial electric networks. The comparative tests performed on an IEEE test system have demonstrated the correctness of the proposed algorithm.

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